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A Comparative Analysis of Exponential and Linear Roundabout Capacity Models Using HCM Research Data

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About this presentation

A summary of an investigation to assess exponential and linear model forms used in practice for roundabout capacity estimation will be presented.

Analyses were carried out using the HCM single-lane roundabout capacity research data (FHWA 2015).

FHWA (2015). Assessment of Roundabout Capacity Models for the Highway Capacity Manual. Accelerating Roundabout Implementation in the United States - Volume II of VII. Publication No. FHWA-SA-15-070. US Department of Transportation, Federal Highway Administration, McLean, Virginia, USA.

Two detailed reports are available for download from http://www.sidrasolutions.com/Resources/Articles

AKÇELIK, R., SHIRKE, C., BESLEY, M., ESPADA, I. and BILLINGHURST, D. (2022). A Comparative Analysis of Exponential and Linear Roundabout Capacity Models Using HCM Research Data. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

AKÇELIK, R. (2022). Searching for a Gap Acceptance Theory Basis for Linear Capacity Models. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

Acknowledgement

We thank Kittelson & Associates for providing the HCM roundabout capacity research data used in this investigation.

Photo: *nearmap*

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Presentation Content

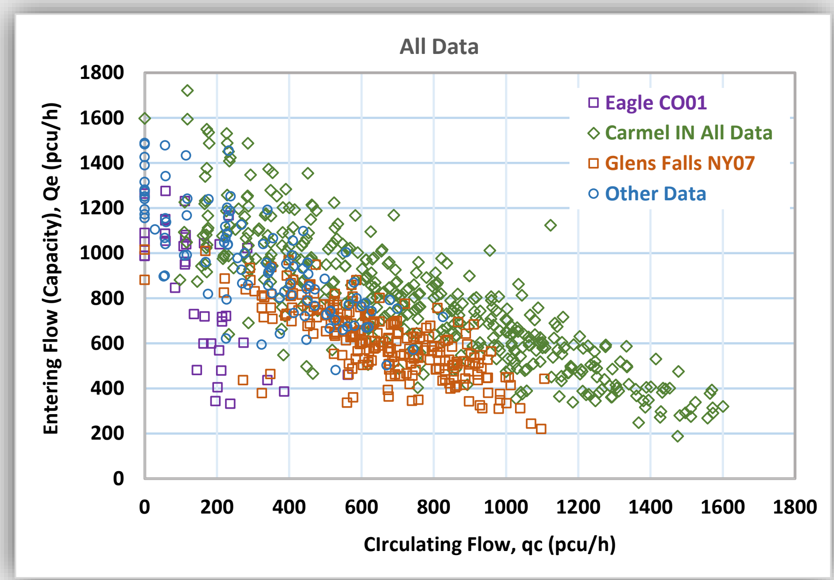
- Analysis method
- Geometry parameters
- Frequency of data points and other aspects of the data
- Best fit and anchored regression results
- Follow-up headway and critical gap (headway)
- Two-segment regression models
- Aggregate data analysis
- Quadratic model
- Calibrated capacity models
- An example to explain unbalanced flow conditions
- Can a linear gap-acceptance capacity model be derived?
- Main conclusions

Analysis method

Assessments focused on the HCM (Siegloch M1) EXPONENTIAL capacity model and the TRL-Kimber LINEAR capacity model. The main focus is on basic model forms.

Only single-lane roundabout data were analysed.

The exponential and linear models were assessed using the full HCM capacity dataset (All Data) and the data subsets for the Glens Falls roundabout and Carmel roundabouts. These data subsets were identified by Johnson and Lin (2018) as having two different roundabout geometry types. These subsets represented horizontal slicing of the HCM capacity data indicating different capacity levels over the same range of circulating flows.



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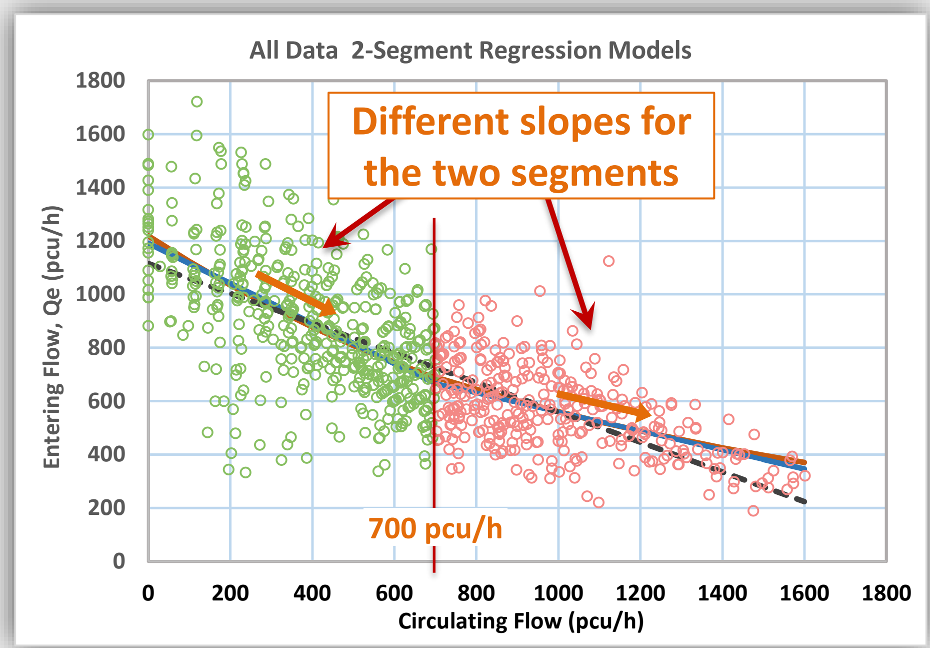
Analysis method

To assess the applicability of models to low and high circulating flow levels, additional data subsets were also considered by vertical slicing of the HCM capacity data into lower and higher sets of circulating flow values. Using this method, two-segment linear and exponential models were analysed.

Different slopes of linear regressions for the two segments show the non-linearity of roundabout capacity data.



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Roundabout capacity models assessed

Roundabout capacity models assessed:

- Basic linear and exponential capacity models derived from
  - best fit regressions
  - regressions with the y-intercept anchored.
- Models that employ average geometry parameters:
  - TRL-Kimber linear capacity model
  - HCM exponential model with the Basic SIDRA Geometry Method added (a simplified version of the SIDRA geometry method).

Exponential model investigated is the Siegloch M1 model as used in the HCM.

It assumes a negative exponential (random) distribution of circulating stream headways.

This differs from the bunched exponential distribution of headways which is the basis of the SIDRA gap-acceptance capacity models.

Roundabout geometry parameters used for analyses

A good amount of judgement is needed in measuring the **entry radius** and **entry angle** values in particular. This may lead to differences in values from measurements by different people.

Dataset		D_i	r_e	ϕ_e	w_L	w_a	L_f
All Data	(ft)	125	47	20°	13	11.6	22
	(m)	38.1	14.3		3.96	3.54	6.7
Glens Falls NY07	(ft)	105	21	26°	12	11	20
	(m)	32.0	6.4		3.66	3.35	6.1
Carmel IN All Data	(ft)	138	65	16°	14	12	23
	(m)	42.1	19.8		4.27	3.66	7.0

D_i = inscribed diameter, r_e = entry radius, ϕ_e = entry angle, w_L = effective entry lane width (smaller than full entry width), w_a = approach half width, L_f = effective flare length

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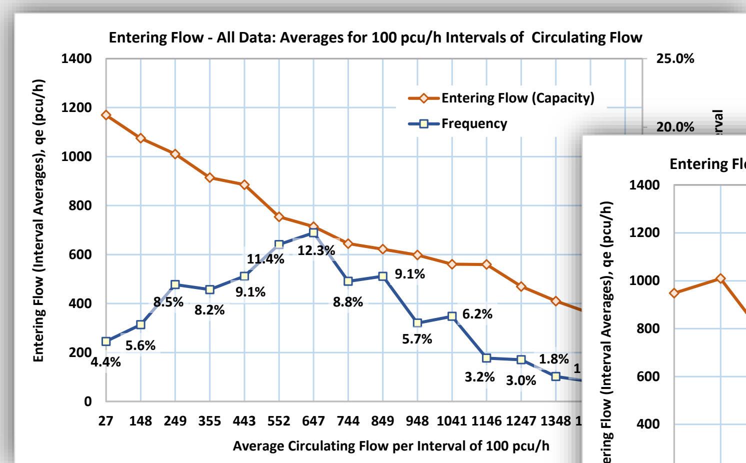
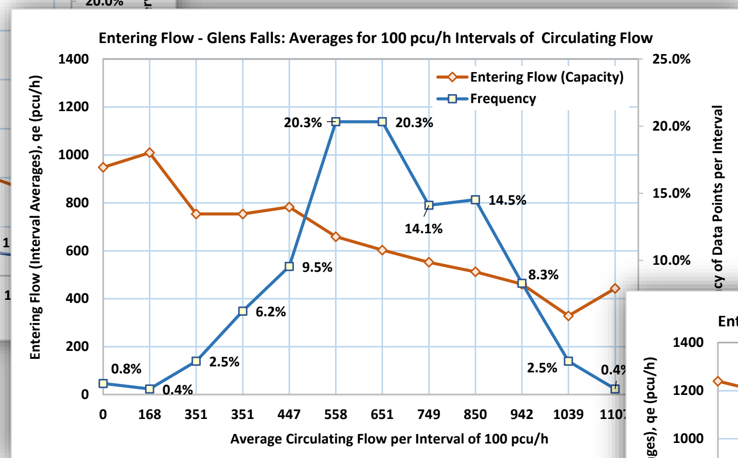
Frequency of data points

All Data

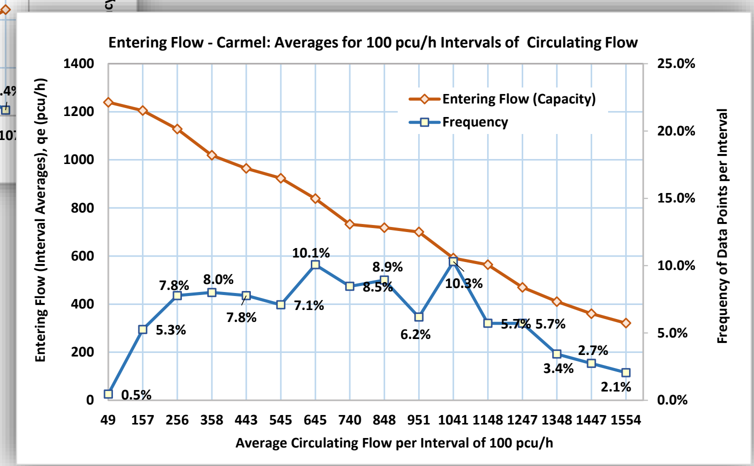


Glens Falls



Note the **more uniform distribution** for Carmel data.

Carmel



Model developers should pay attention to the effect of the **frequency of data points** by circulating flow on best fit regression results for the linear and nonlinear models as **this is likely to cause a bias towards hiding non-linearity** of the capacity curve.

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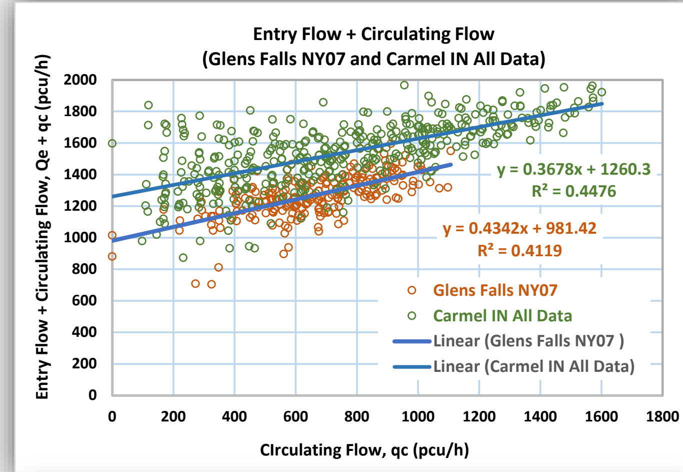
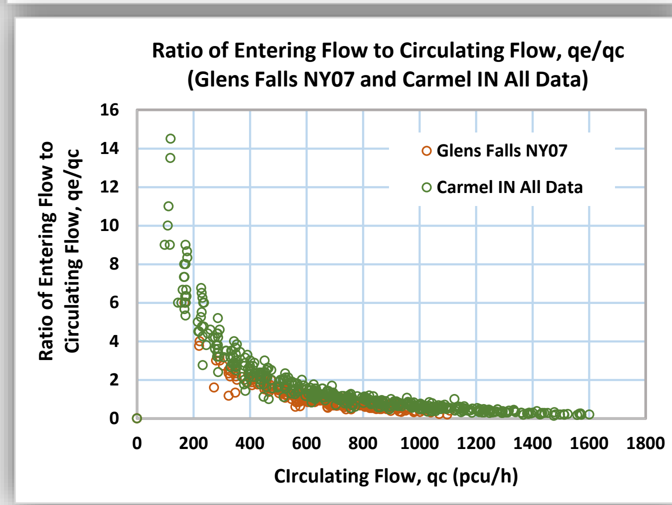
Other aspects of data

High values of the sum of entering flow (capacity) and circulating flow are achieved at high circulating flows. This may indicate a potential for higher entry flow values at low circulating flows.



Sum of Entering Flow (Capacity) and Circulating Flow

Ratio of Entering Flow to Circulating Flow relates to the modelling of unbalanced flow conditions (used in the SIDRA model).



Ratio of Entering Flow to Circulating Flow

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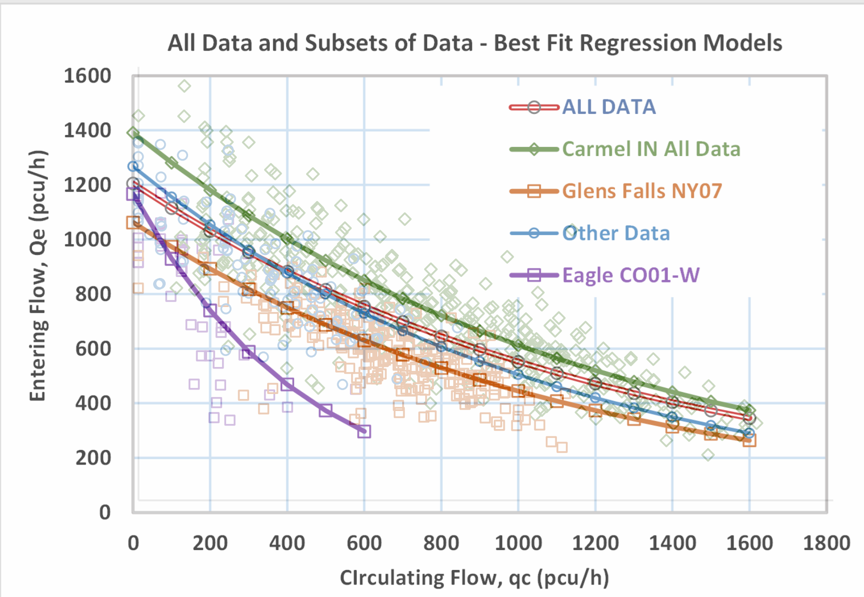
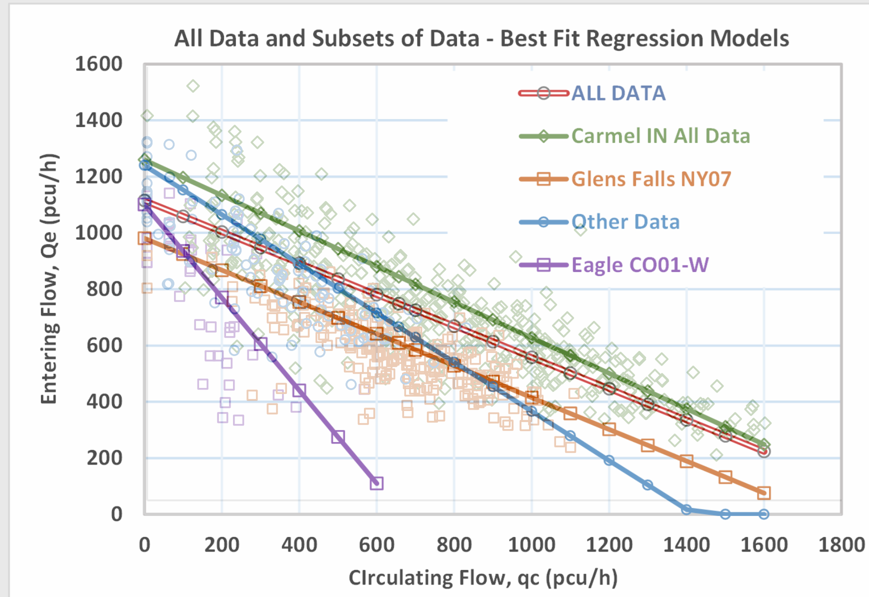
Best fit regression models

Eagle CO01-W site: outlier

Exponential



Linear



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Eagle CO01-W site: outlier

The Eagle CO01 site indicates outlier characteristics which could be explained by the conditions of this site. Excluding this site from analyses affected the results.

However, the analyses using All Data were carried out including the Eagle Site data as in the HCM model development.

Regression type	A	B	t_f	t_c	RMSE
All Data with Eagle CO01					
Exponential (Siegloch M1)	1205	0.00078	3.00	4.30	180
Linear	1115	- 0.5570	3.23	-	184
All Data without Eagle CO01					
Exponential (Siegloch M1)	1271	0.00085	2.83	4.48	168
Linear	1148	- 0.5916	3.14	-	174

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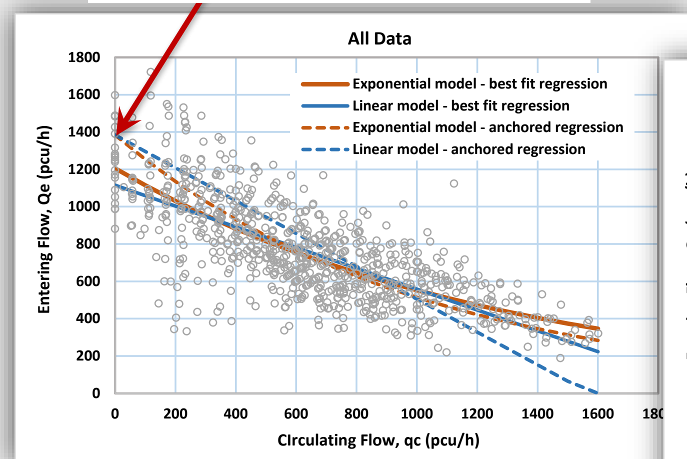
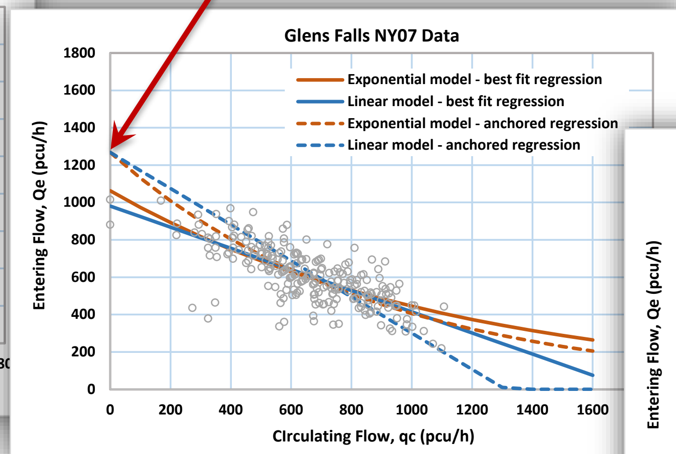
Best fit and anchored regression models

Anchored regressions were conducted by specifying the **y-intercept (A)** values based on the measured follow-up headways, t_f ($A = 3600 / t_f$).

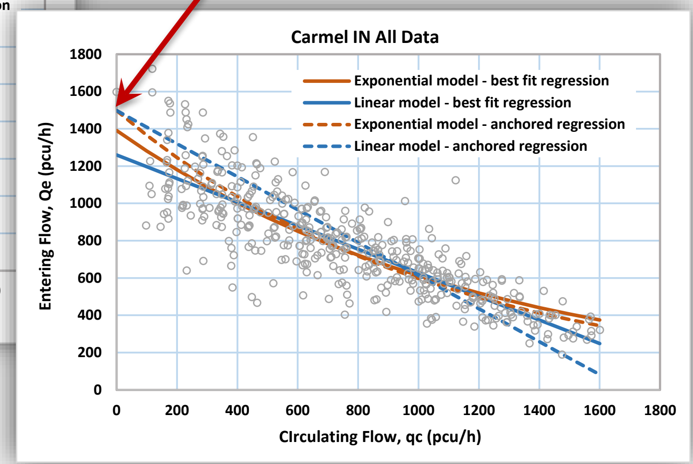
t_f = 2.601 s, A = 1384 pcu/h



Anchored: t_f = 2.838 s, A = 1268 pcu/h



t_f = 2.405 s, A = 1497 pcu/h



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Best fit and anchored regression models

The capacity estimates from anchored regressions indicate that the exponential model estimates can stay close to the best fit regression estimates for medium to high circulating flows whereas the linear model capacity estimates become significantly lower at high circulating flows. This is due to the constant slope of the linear model.

The reducing slope of the exponential model helps it to adopt to the changes in the observed data. Results show small increases in RMSE values for the anchored regressions for the exponential model (2.0% to 6.7%) but large increases for the linear model (21.4% to 28.7%).

	A	B	t_f	t_c	t_f / t_c	RMSE	
Exponential (Siegloch M1)	1205	0.00078	2.988	4.302	0.695	180	
Basic Linear	1115	-0.5570	3.229	-	-	184	
A (t_f) parameter anchored							Increase in RMSE
Exponential (Siegloch M1)	1384	0.00099	2.601	4.865	0.535	191	5.7%
Basic Linear	1384	-0.8795	2.601	-	-	224	22.1%

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Follow-up headway and critical gap values implied by best fit regressions

The best fit regression models are found to imply larger values of follow-up headways compared with the measured follow-up headways (t_f).

The survey methods used for these gap acceptance parameters should be paid more attention. The Siegloch survey method is recommended since it measures critical gaps and follow-up headways at the same time. Ideally, the follow-up headway and critical gap surveys should also be carried out at the same time as the capacity surveys.

	Exponential (Siegloch M1) Model			Linear Model		
Capacity Data >>	ALL DATA	Glens Falls NY07	Carmel IN All Data	ALL DATA	Glens Falls NY07	Carmel IN All Data
Regression A =	1205	1062	1391	1115	981	1260
Regression t_f = 3600 / A	2.988	3.390	2.588	3.229	3.670	2.857
Measured t_f =	2.601	2.838	2.405	2.601	2.838	2.405
A = 3600 / t_f =	1384	1268	1497	1384	1268	1497
Ratio of implied t_f to measured t_f	1.15	1.19	1.08	1.24	1.29	1.19
Ratio of implied t_c to measured t_c	0.92	1.01	1.13	-	-	-

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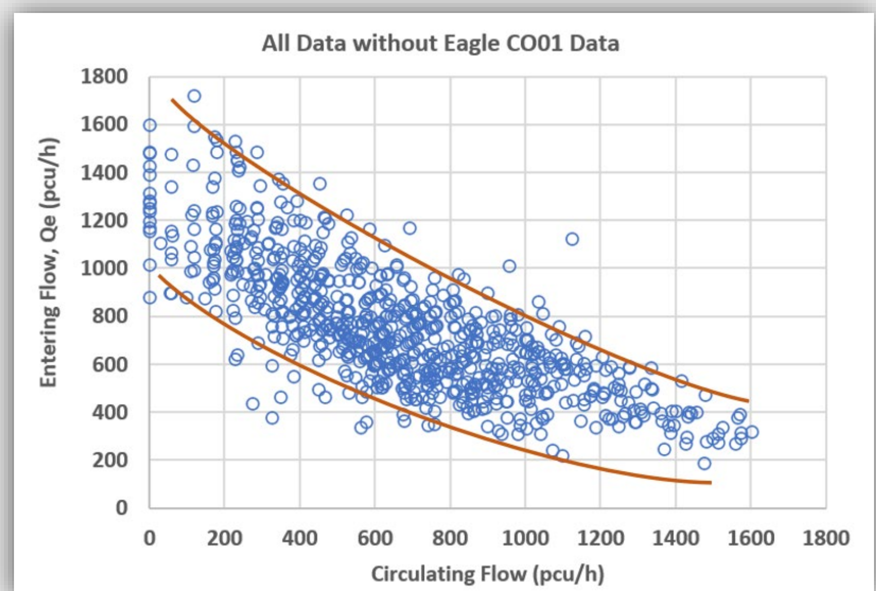
Two-segment regressions

To demonstrate the fundamental exponential (non-linear) characteristic of roundabout capacity, two-segment linear and exponential models were explored via best fit regressions applied to two separate segments (subsets) created by vertical slicing of data.

Analyses were carried out for All Data (including Eagle CO01) and Carmel data.

The Glens Falls dataset was not used for this analysis due to a very low number of data points in the low circulating flow range.



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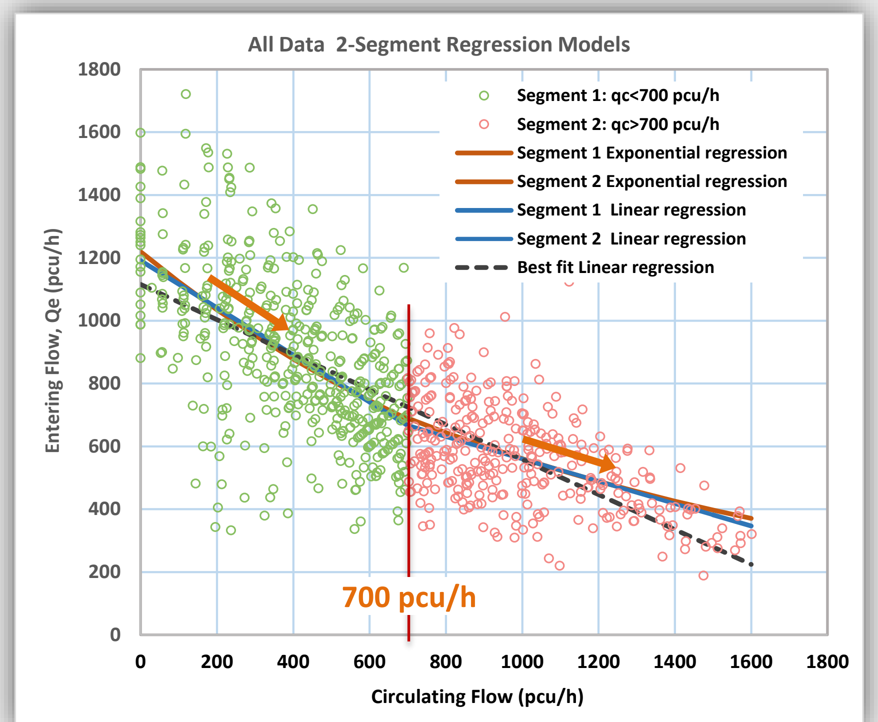
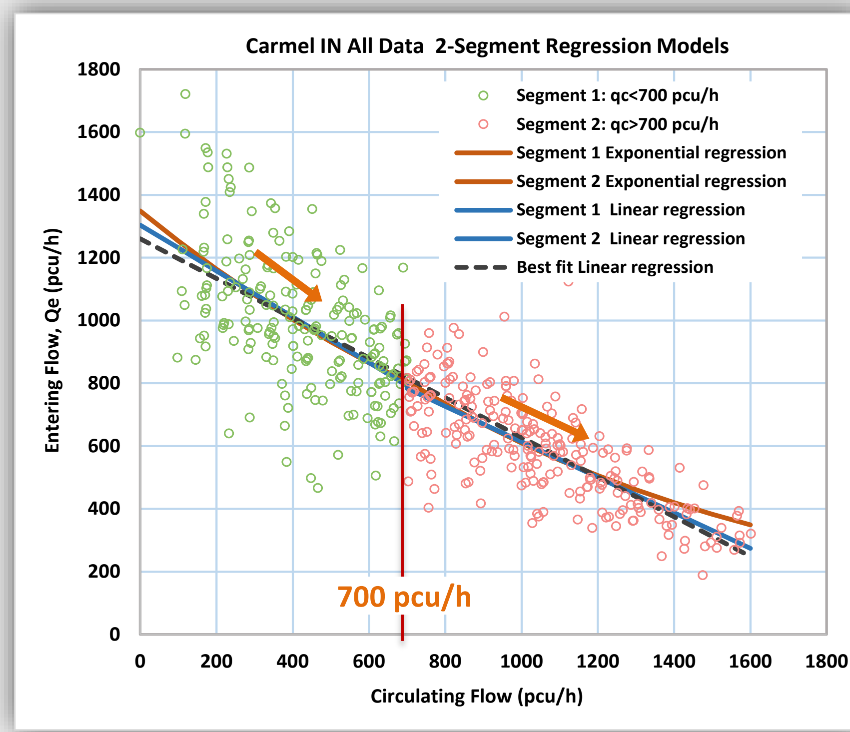
Two-segment best fit regressions for Basic Exponential and Basic Linear capacity models

Two-segment best fit regression results show different slopes and y-intercepts for the linear model. This is because a two-segment linear model has the ability to adapt to differences between low and high circulating flow conditions.





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Two-segment best fit regressions for Basic Exponential and Basic Linear capacity models

Best fit regressions results for All Data

Different slopes and y-intercepts

Basic Linear model

	Aver. q_c	Aver. Q_e	A	B	t_f	t_c	t_f / t_c	RMSE
Segment 1 ($q_c \leq 700$ pcu/h)	408	885	1191	-0.7500	3.023	-	-	180
Segment 2 ($q_c > 700$ pcu/h)	986	571	916	-0.3561	3.930	-	-	
Single segment regression	642	758	1115	-0.5570	3.229	-	-	184

Exponential (Siegloch M1) model

Exponential (Siegloch M1)	Aver. q_c	Aver. Q_e	A	B	t_f	t_c	t_f / t_c	RMSE
Segment 1 ($q_c \leq 700$ pcu/h)	408	885	1219	0.00082	2.953	4.429	0.67	180
Segment 2 ($q_c > 700$ pcu/h)	986	571	1118	0.00069	3.220	4.094	0.79	
Single segment regression	642	758	1205	0.00078	2.988	4.302	0.70	180

q_c: Circulating flow (pcu/h), Q_e = Entering flow (pcu/h), t_f = Follow-up headway (s), t_c = Critical gap (s)

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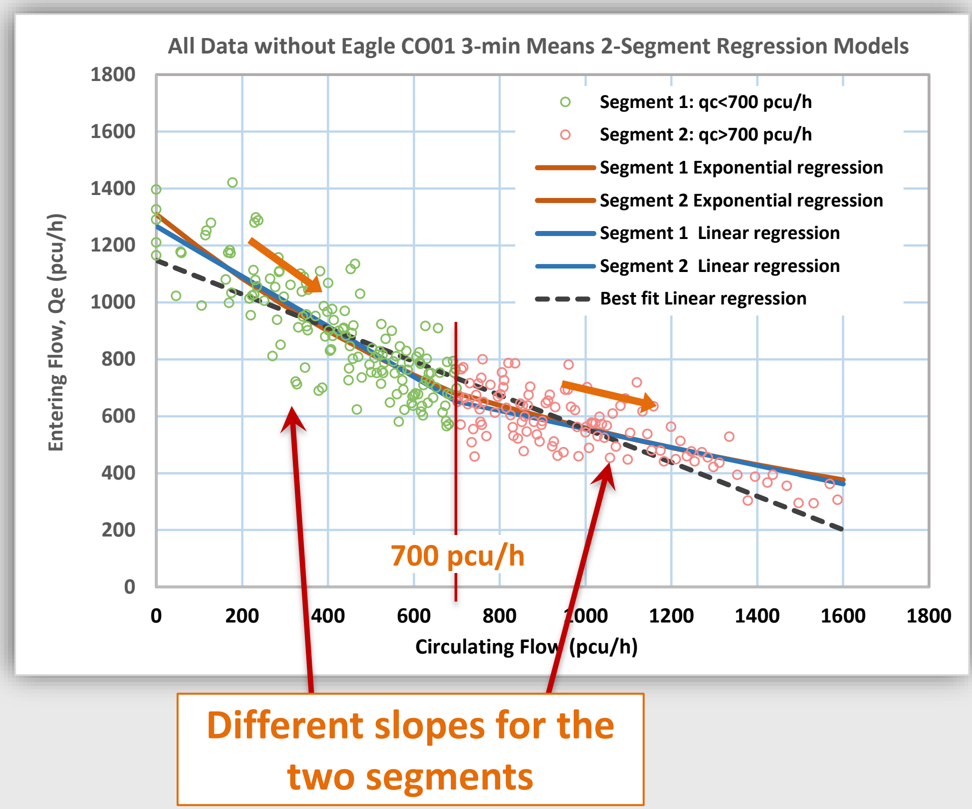
Aggregate (grouped) data analysis

The mean values of entering flow and circulating flow were calculated **for groups of 2 and 3 data points** for testing the best fit and anchored regression models for the basic exponential and linear models with aggregate data (referred to as **Two-Minute and Three-Minute datasets**).

Two-segment regressions were also applied.

These regression were applied **without the Eagle CO01-W site** (outlier).

Results for the **Three-Minute dataset** are given here.



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Aggregate (grouped) data analysis for All Data without Eagle CO01 site

Best fit regressions using Three-Minute dataset

Different slopes and y-intercepts

Basic Linear model

Single-segment analysis	Aver. q_c	Aver. Q_e	A	B	t_f	t_c	t_f / t_c	RMSE
	667	753	1147	-0.5914	3.139	-	-	109
Two-segment analysis								
Segment 1 ($q_c \leq 700$ pcu/h)	430	889	1267	-0.8799	2.841	-	-	97
Segment 2 ($q_c > 700$ pcu/h)	988	570	876	-0.3213	4.110	-	-	

Exponential (Siegloch M1) model

Single-segment analysis	Aver. q_c	Aver. Q_e	A	B	t_f	t_c	t_f / t_c	RMSE
	667	753	1271	0.00084	2.832	4.440	0.638	99
Two-segment analysis								
Segment 1 ($q_c \leq 700$ pcu/h)	430	889	1309	0.00094	2.750	4.759	0.578	98
Segment 2 ($q_c > 700$ pcu/h)	988	570	1081	0.00066	3.330	4.041	0.824	

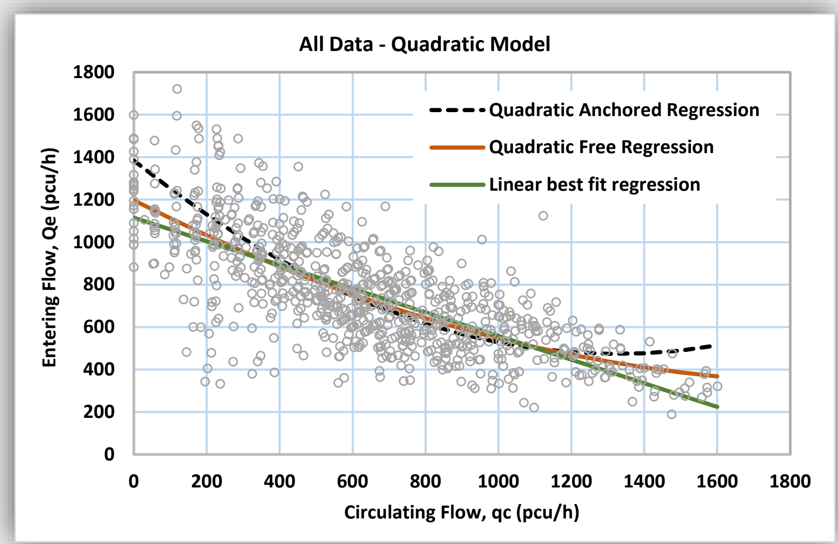
q_c: Circulating flow (pcu/h), Q_e = Entering flow (pcu/h), t_f = Follow-up headway (s), t_c = Critical gap (s)

Quadratic model

This **quadratic (second degree polynomial)** model was tested by Kimber (1980) for estimating capacity as a function of circulating flow:

$$Q_e = A + B\, q_c + C\, q_c^2$$

A poor characteristic of this model is that capacity estimates start increasing at high circulating flows as seen clearly for the anchored regression model. Therefore, **the quadratic model is not recommended for use.**



Best fit regression	A	B	C	t_f	t_c	t_f / t_c	RMSE
Basic Quadratic	1197	-0.8700	0.00022	3.008	-	-	180
Basic Linear	1115	-0.5570	-	3.229	-	-	184
Anchored regression	**A**	**B**		**t_f**	**t_c**	**t_f / t_c**	**RMSE**
Basic Quadratic	1384	-1.376	0.00052	2.601	-	-	190
Basic Linear	1384	-0.8795	-	2.601	-	-	224

Calibrated capacity models

Alternative model calibration methods were applied to the **HCM (Siegloch M1) Exponential model** with a new **Basic SIDRA Geometry Method** added and the **TRL-Kimber Linear model** with geometry parameters.

The original estimates from these models are referred to as **default models**.

Basic SIDRA Geometry Method

Follow-up headway: $t_f = f_e \, f_a \, f_r \, t_f'$

Critical gap (headway): $t_c = 1.8 \, t_f$

Average roundabout geometry parameters were used in the analyses.

Environment (Calibration) Factor, default: $f_e = 1.05$

Entry Angle Adjustment Factor: $f_a = 0.94 + 0.000026 \, \phi_e^{1.6}$

Entry Radius Adjustment Factor: $f_r = 0.95 + 3.28 \, / \, r_e$

Unadjusted Follow-up Headway (seconds): $t_f' = 3.18 - 0.0061 \, D_i + 7.8\text{x}10^{-6}$

ϕ_e: entry angle (degrees), r_e: entry radius (feet), D_i : inscribed diameter (feet).

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Calibration methods

Calibration Methods are described in detail in Akcelik (2022).

TRL-Kimber linear model (Kimber 1980)

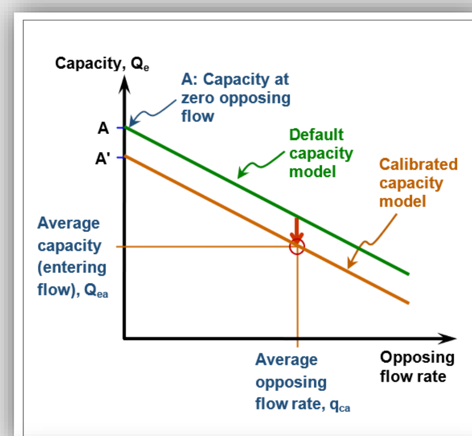
$$Q_e = A + B\,q_c$$

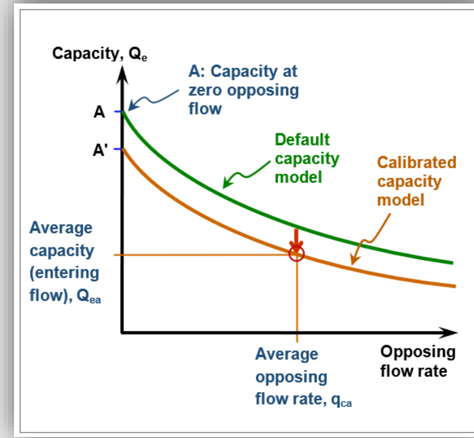
- Method 1: Use B from the default model used to determine A.
- Method 2: Use A from the default model used to determine B.
- Method 3: Use $A = 3600 / t_f$ using measured t_f to determine B.

HCM (Siegloch M1) Exponential model (TRB 2016)

$$Q_e = A\,exp\,(-B\,q_c)$$

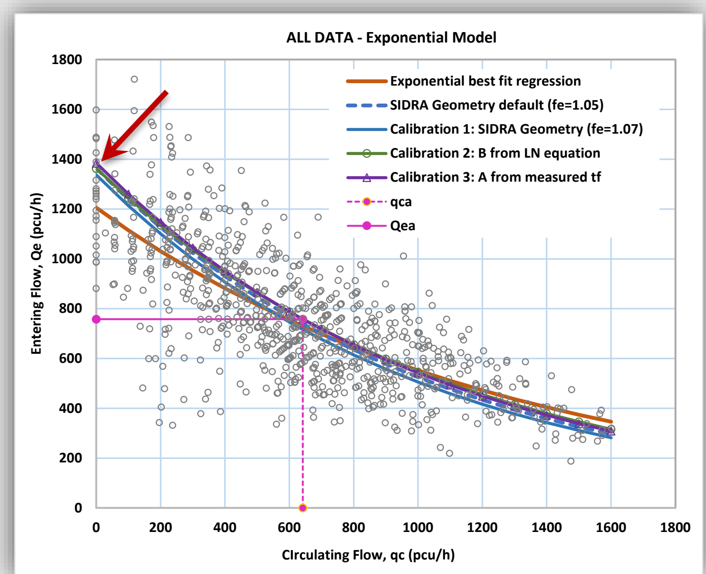
- Method 1: Adjust Environment (Calibration) Factor, f_e.
- Method 2: Use A from the default model used to determine B.
- Method 3: Use $A = 3600 / t_f$ using measured t_f to determine B.



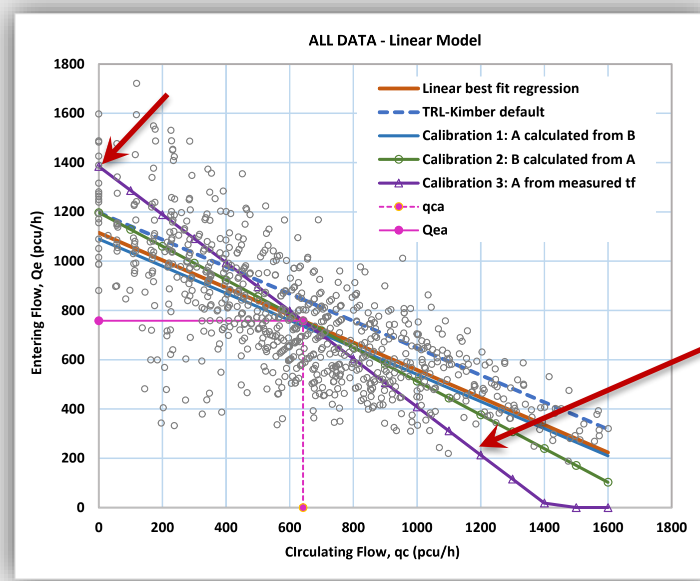


Calibrated models

HCM Exponential model with the Basic SIDRA Geometry Method



TRL-Kimber Linear model



Calibration using the measured follow-up headway has difficulty (similar to anchored regression models)

- Calibration Methods 1 and 2 are seen to be close to best fit regression models.
- The RMSE values for Calibration Method 3 are reasonably close to best fit regression values for the HCM model with the Basic SIDRA Geometry Method (4.3% to 7.1%). Differences are large for the TRL-Kimber model (26.0% to 31.0%).

Calibrated models

- Large increases in RMSE values for Calibration Method 3 (using the measured follow-up headway) are seen for the TRL-Kimber model (26.0% to 31.0%). This is similar to the results of anchored regressions for the basic linear model (21.4% to 28.7%).

Exponential model with SIDRA Geometry	A	B	t_f	t_c	t_f / t_c	RMSE	Increase in RMSE
Basic Exponential - Best fit regression	1205	0.000780	2.988	4.302	0.695	180	-
SIDRA Geometry default, f_e = 1.05	1363	0.000954	2.641	4.755	0.556	188	4.6%
Calibration 1: f_e = 1.07	1337	0.000972	2.693	4.845	0.556	187	3.6%
Calibration 2: B from A	1363	0.000914	2.641	4.612	0.573	190	5.3%
Calibration 3: B from A using measured t_f	1384	0.000938	2.601	4.678	0.556	192	6.5%

TRL-Kimber Linear model	A	B	t_f	t_c	t_f / t_c	RMSE	Increase in RMSE
Basic Linear - Best fit regression	1115	-0.5570	3.229	-	-	184	-
TRL-Kimber default model	1197	-0.5495	3.008	-	-	203	10.5%
Calibration 1: A from B	1090	-0.5495	3.304	-	-	185	0.7%
Calibration 2: B from A	1197	-0.6840	3.008	-	-	189	2.9%
Calibration 3: B from A using measured t_f	1384	-0.9755	2.601	-	-	235	28.0%

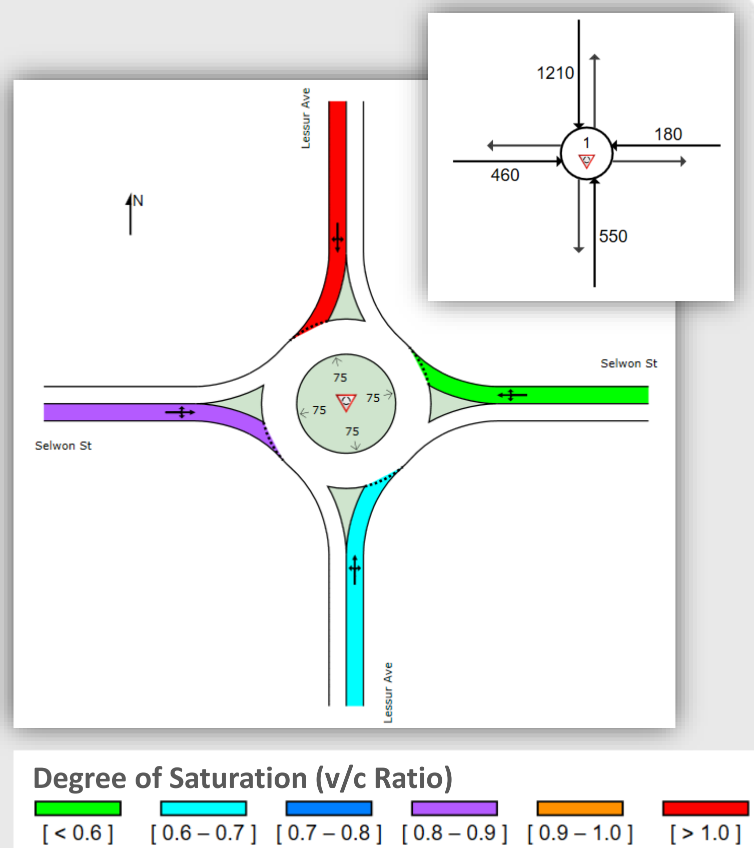
Average entering flow (capacity), Q_{ea} = 758 pcu/h, Average circulating flow, q_c = 642 (pcu/h)

Example for modeling unbalanced flow conditions

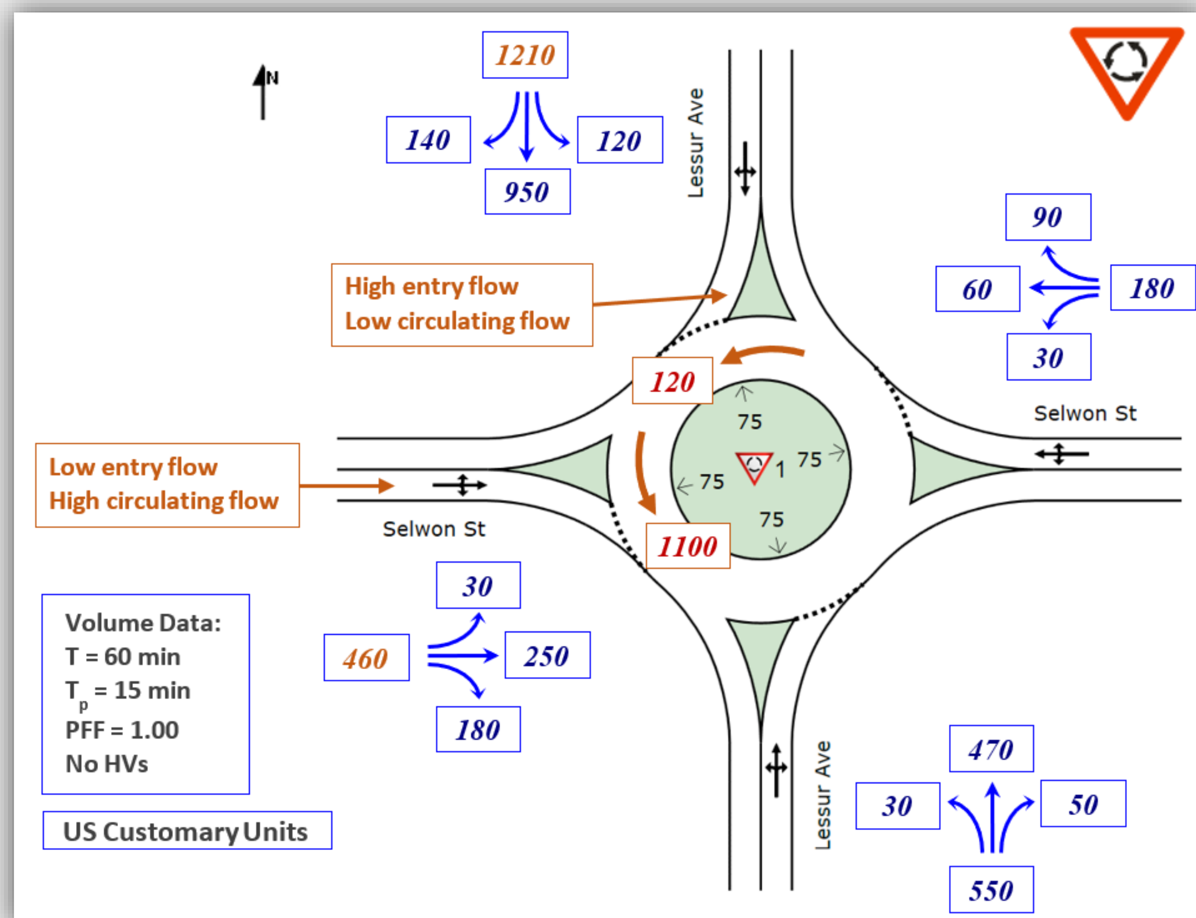
The statistical error levels (RMSE) are given for all models tested. While statistical error levels are important, model choices should not be based only on statistical error levels of available data.

It is emphasised that models should also be assessed in terms of ability to deal with specific situations. Demand flow patterns causing unbalanced flow conditions at high demand flows is one of these.

A single-lane roundabout example is given for unbalanced flow conditions in order to explain the interactions among roundabout entry flows from different approaches causing these conditions. The concern about linear roundabout capacity models underestimating capacity for low circulating flows is relevant to these specific conditions.



Example for modeling unbalanced flow conditions



Roundabout geometry parameters

	D_i	r_e	f_e	w_L	w_a	L_f
(ft)	125	47	20°	13	11.6	22
(m)	38.1	14.3		3.96	3.54	6.7

D_i = inscribed diameter
r_e = entry radius
ϕ_e = entry angle
w_L = effective entry lane width (smaller than full entry width),
w_a = approach half width,
L_f = effective flare length

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Example for modeling unbalanced flow conditions

Approach	Demand Flow	Circulating Flow	Capacity	v/c Ratio (Deg. Of Saturation)	Delay	Level of Service	95% Back of Queue
	veh/h	pcu/h	veh/h		sec		vehicles
HCM 6 model parameters based on BEST FIT Exponential Regression (A = 1205, B = 0.00078, t_f = 2.99, t_c = 4.30) * Capacity Constraint applied to North approach							
North (SB)	1210	120	1097	1.103	67.9	LOS F	93.2
West (EB)	460	1000 *	552	0.833	34.7	LOS D	6.5
South (NB)	550	389 *	890	0.618	13.3	LOS B	5.5
East (WB)	180	530	797	0.226	6.9	LOS A	0.7
HCM 6 model parameters based on ANCHORED Exponential Regression (A = 1384, B = 0.00099, t_f = 2.60, t_c = 4.87) Capacity Constraint did not apply to North approach							
North (SB)	1210	120	1229	0.985	34.4	LOS D	62.9
West (EB)	460	1100	466	0.988	67.2	LOS F	13.3
South (NB)	550	400	931	0.590	12.1	LOS B	5.6
East (WB)	180	530	819	0.220	6.7	LOS A	1.0

Roundabout LOS Method: Same as Sign Control

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Example for modeling unbalanced flow conditions

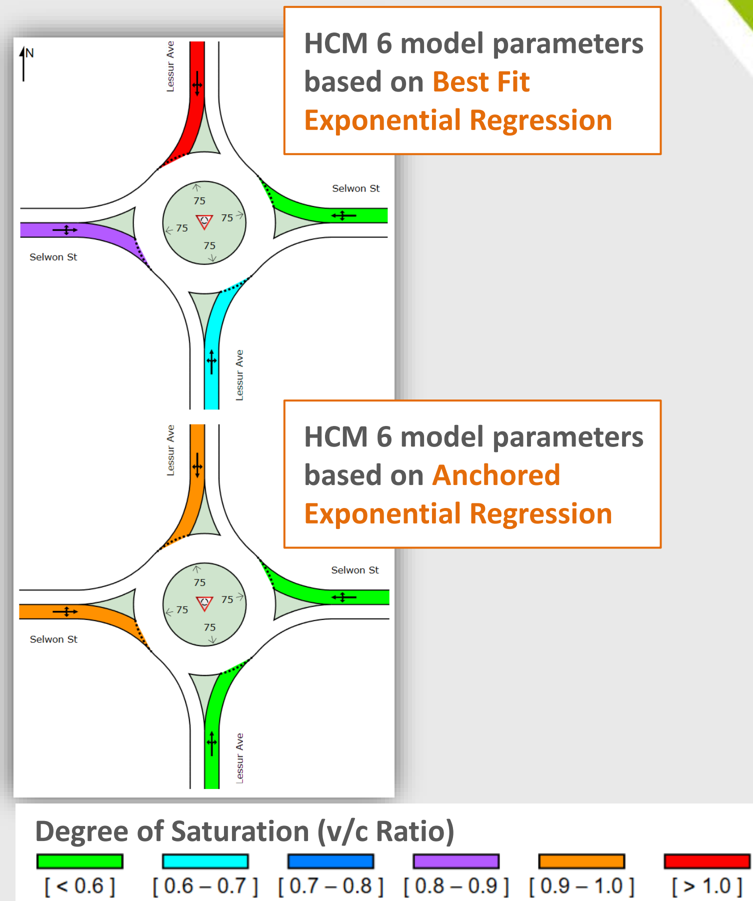
The anchored regression model used for the HCM Edition 6 roundabout capacity model is supported as it is useful in modelling specific cases of unbalanced flow conditions.

This model estimates higher capacities at low circulating flows as it uses measured follow-up headways that correspond to y-intercept (capacity) values which are larger than those estimated by the best fit regression models.

The example is set to explain how the best fit regression model (with lower RMSE values based on general data)

- fails to estimate the effect of unbalanced flow conditions and
- overestimates the delay and LOS on the North approach.

Applying the SIDRA Model for Unbalanced Flow Conditions indicated worse performance for the West approach.



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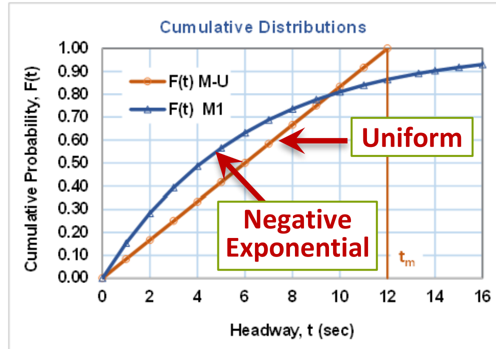
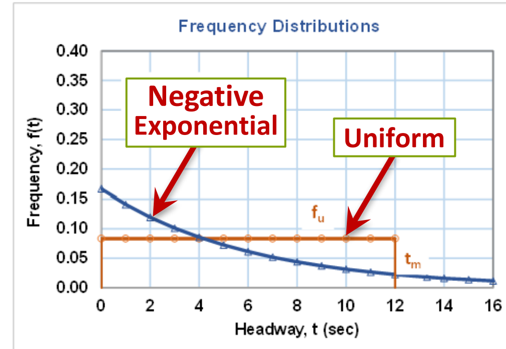
Can a linear gap-acceptance capacity model be derived?

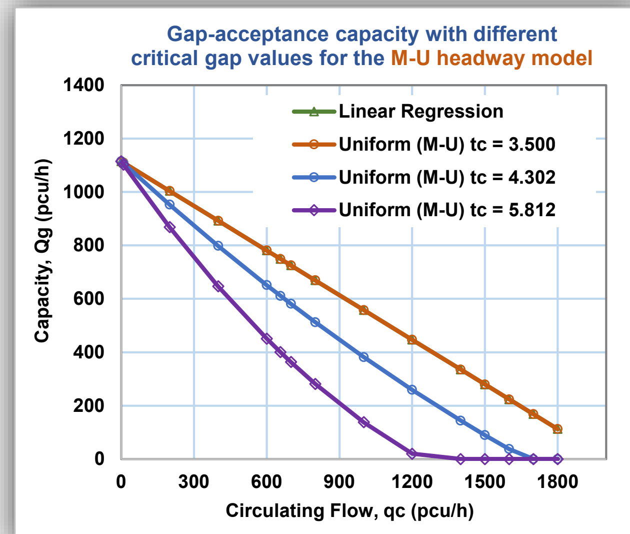
Since most **gap-acceptance capacity models** use **exponential forms of arrival headway distribution**, the resulting capacity models have an exponential form.

A theoretical investigation was carried out to explore if a **linear capacity model** can be derived as a **gap-acceptance capacity model**. **Uniform and linear headway distributions** of circulating flows were assumed although these are **not realistic** given the **random nature of arrival headways** including **bunching** considerations.

The investigation concluded that both uniform and linear headway distributions resulted in **non-linear gap-acceptance capacity models** with **unrealistic features**.







Refer to:

AKÇELIK, R. (2022). **Searching for a Gap Acceptance Theory Basis for Linear Capacity Models**. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

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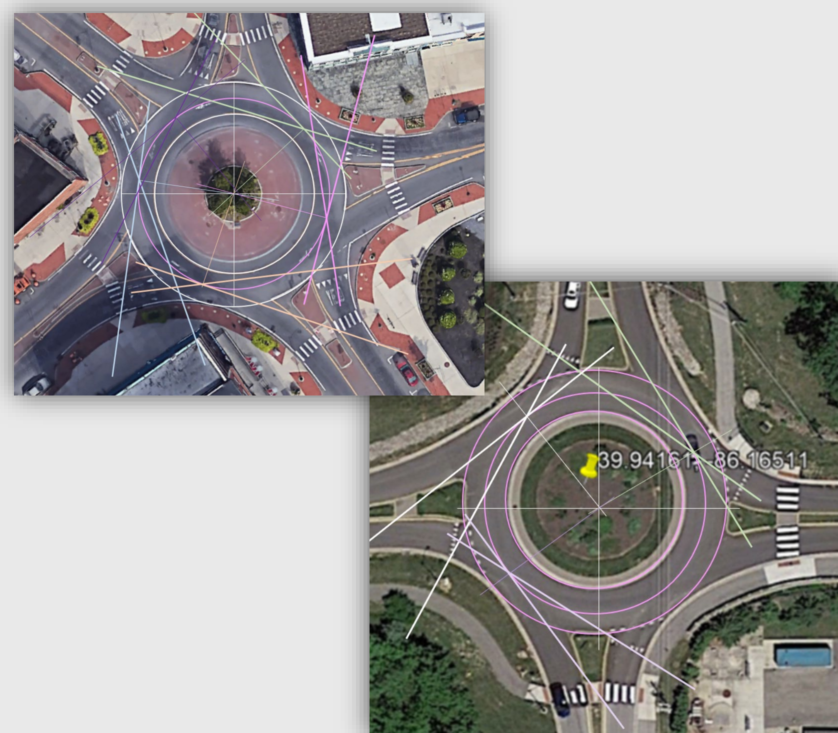
Main conclusions - 1

Roundabout geometry types

Analyses of calibration methods for subsets of data using both the **HCM (Siegloch) exponential capacity model with the Basic SIDRA Geometry Method added** and the **TRL-Kimber model** supported the finding by Johnson and Lin (2018) that roundabout geometry parameters may have a combined (aggregate) effect on capacity of different **roundabout geometry types**.



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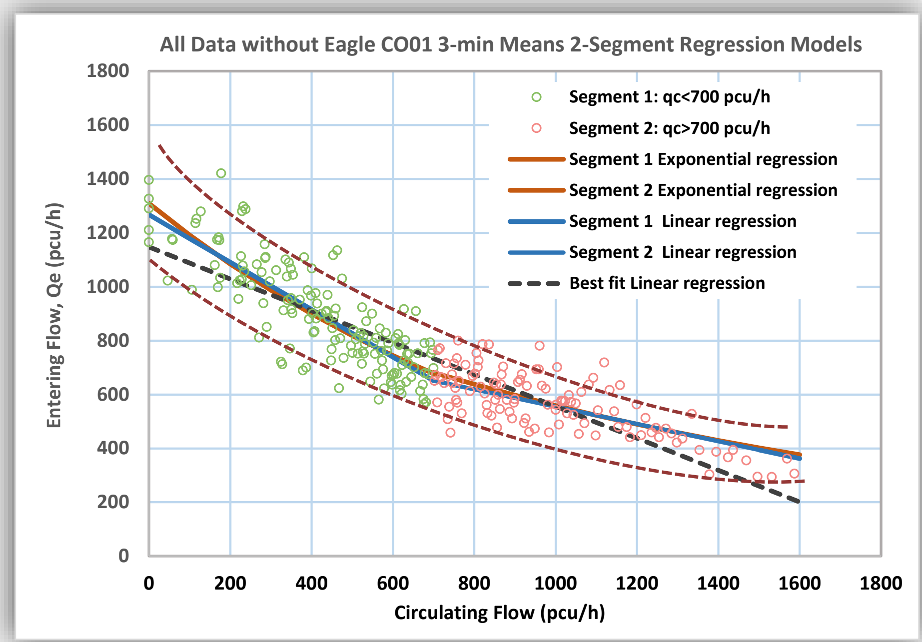
Main conclusions - 2

Our preferred model

The assessments from various perspectives conducted using the HCM single-lane roundabout capacity research data reported in our reports demonstrate the non-linear characteristic of roundabout capacity data.

They are found to support the HCM exponential (non-linear) roundabout capacity model over the linear model form which has shortcomings in estimating capacity at low and high circulating flows.



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END OF PRESENTATION

Thank you!

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